Advanced Topics in General Insurance

Exam GIADV

Date: Tuesday, November 3, 2020

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 8 questions numbered 1 through 8 with a total of 40 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Written-Answer Instructions

1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.

   a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 (and ^ used to indicate a superscript).

   b) In the Excel document formulas should be entered. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit.

   c) Individual exams may provide additional directions that apply throughout the exam or to individual items.

2. The answer should be confined to the question as set.

3. The Word and Excel files that contain your answers must be uploaded before time expires.

1. (*5 points*) Casualty Reinsurance Company is writing a casualty per occurrence excess treaty for accident year 2021 covering the layer 750,000 excess of 250,000.

You are given the following information:

Loss Experience Evaluated as of December 31, 2019		
Accident Date	**Untrended Loss**	**Untrended ALAE**
July 1, 2017	200,000	150,000
July 1, 2017	350,000	400,000
July 1, 2018	225,000	0
July 1, 2018	900,000	450,000
July 1, 2019	250,000	50,000
July 1, 2019	800,000	275,000

- All losses of at least 200,000 are shown.
- All policy limits throughout the experience period are 1,000,000 and are expected to remain at this level through 2021.
- On level subject premium is 10,000,000 for each year from 2017-2019.
- ALAE is allocated to layer in proportion to losses.
- Loss and ALAE trend are each 5% per year.
- The following accident year development factors are applicable to both loss and ALAE in the layer 750,000 excess of 250,000:

12-Ultimate	2.40
24-Ultimate	1.40
36-Ultimate	1.10

Estimate the experience rating loss cost, including ALAE, as a percentage of the subject premium.

> *The response for this question is to be provided in the Excel spreadsheet.*

2. (*4 points*) You are using the Capital Asset Pricing Model (CAPM) to determine an underwriting profit margin.

You are given the following information about an insurer's homeowners line of business:

- The risk-free rate is 2%.
- The expected return on the market portfolio is 10%.
- The liability beta is –0.2.
- The ratio of premium to owner's equity is 2 to 1.
- The tax rate on underwriting income is 30%.

The homeowners line of business has three distinct payment pattern groups:

Group	Percentage of the Insurer's Business	Average Time Between Receipt of Premium and Payment of Losses and Expenses
1	40%	0.9 years
2	35%	1.2 years
3	25%	1.5 years

The investment portfolio backing the homeowners line of business has the following characteristics:

Asset	Percentage of Total Assets	Tax Rate
Tax exempt bonds	20%	0%
Corporate dividend income stocks	30%	10%
Taxable bonds	50%	30%

Responses for all parts of this question are to be provided in the Excel spreadsheet.

(a) (*0.5 points*) Calculate the funds generating coefficient estimate, k.

(b) (*0.5 points*) Calculate the underwriting beta.

(c) (*1.5 points*) Calculate the underwriting profit margin ignoring taxes.

(d) (*1.5 points*) Calculate the underwriting profit margin using the version of CAPM that accounts for taxes.

3. (*5 points*) You are given the following data extracted from a triangle of cumulative paid losses:

Accident Year	From (months)	To (months)	Increment	Diagonal Age	Accident Year Total
2016	0	12	2,500	48	5,000
2016	12	24	1,800	48	5,000
2016	24	36	500	48	5,000
2016	36	48	200	48	5,000
2017	0	12	4,100	36	7,000
2017	12	24	2,000	36	7,000
2017	24	36	900	36	7,000
2018	0	12	4,600	24	6,800
2018	12	24	2,200	24	6,800
2019	0	12	5,300	12	5,300

You are also given the following onlevel premiums:

Accident Year	Onlevel Premium
2016	10,000
2017	12,000
2018	15,000
2019	18,000

You apply Clark's stochastic reserving model using the Cape Cod method and a loglogistic distribution with cumulative distribution function $G(x) = \dfrac{x^{\omega}}{x^{\omega} + \theta^{\omega}}$ where x is in months.

In Clark's model, the distribution of ultimate reserves is approximated by a discrete distribution.

(a)   (*0.5 points*) Explain why this should not be a cause for concern.

ANSWER:

Maximum likelihood estimates can be obtained by maximizing the function $\ell = \sum_{i}\left[c_i \ln(\mu_i) - \mu_i \right]$. The maximum likelihood estimate of θ is 6.7805, of ω is 0.9993, and of *ELR* is 0.6386.

3. Continued

(b) (*2 points*) Calculate the value of ℓ at its maximum.

> *The response for part (b) is to be provided in the Excel spreadsheet.*

(c) (*1 point*) Estimate the scale factor, σ^2.

> *The response for part (c) is to be provided in the Excel spreadsheet.*

(d) (*1 point*) Create a scatter plot in which the x values are the expected incremental losses and the y values are the normalized residuals.

> *The response for part (d) is to be provided in the Excel spreadsheet.*

(e) (*0.5 points*) Interpret the scatter plot in part (d) with regard to determining if the model assumptions are correct.

> ANSWER:

4. (*9 points*) You are interested in determining the variability of unpaid claim estimates. The triangle of paid claims data you are working with, by accident year (AY) and development year, is presented below. It is assumed that all claims are fully developed after seven years.

Mack's method of estimating reserve variability is to be applied to this triangle.

AY	Development Year							Standard Error
---	1	2	3	4	5	6	7	
1	5,012	8,269	10,907	11,805	13,539	16,181	18,009	0
2	106	4,285	5,396	10,666	13,782	15,599		559
3	3,410	8,992	13,873	16,141	18,735			
4	5,655	11,555	15,766	21,266				
5	1,092	9,565	15,836					7,865
6	1,513	6,445						6,234
7	557	1,629						14,344
	Age-to-Age Factors							
1	1.650							
2	40.425							
3	2.637							
4	2.043							
5	8.759							
6	4.260							
f_k	2.925							
α_k^2	40,350							

(a) (*1.5 points*) State the three statistical assumptions underlying the chain ladder model.

ANSWER:

(b) (*1 point*) Complete the triangle of age-to-age factors.

The response for part (b) is to be provided in the Excel spreadsheet.

(c) (*1.5 points*) Calculate the remaining values of f_k and α_k^2.

The response for part (c) is to be provided in the Excel spreadsheet.

4. Continued

(d)   (*1 point*) Square the development triangle by completing the remaining shaded cells, where one calculated value is provided.

> *The response for part (d) is to be provided in the Excel spreadsheet.*

(e)   (*3 points*) Calculate the remaining standard errors of the reserve estimators for the individual accident years.

> *The response for part (e) is to be provided in the Excel spreadsheet.*

An alternative to the chain ladder model is the parameterized BF model.

(f)   (*1 point*) Describe how expected future emergence differs between the two models.

> ANSWER:

5. (*4 points*) Big Reinsurance Company (BRC) is renewing two accounts, X and Y, each of which is exposed to three independent claim events, 1, 2 and 3. You are given the following information:

Event (*i*)		Loss for Account	
i	*p(i)*	X	Y
1	1%	20,000	5,000
2	2%	10,000	8,000
3	4%	5,000	2,000

- *p(i)* represents the probability of Event *i*.
- The risk load multiplier, λ, is 0.000024.

Responses for all parts of this question are to be provided in the Excel spreadsheet.

(a) (*2 points*) Calculate the renewal risk load for each account using the Marginal Variance method.

(b) (*0.5 points*) Demonstrate that the Marginal Variance method is not renewal additive.

BRC is considering using the Covariance Share method to calculate risk loads. The shared covariance of each event will be allocated to each account in proportion to its loss for that event.

(c) (*1.5 points*) Calculate the risk load for each account using the Covariance Share method.

6. (*4 points*) You are calculating a risk margin for outstanding claim liabilities as discussed in "A Framework for Assessing Risk Margins." There are two lines of business, motor and home.

There are two sources of systemic risk: internal risk and external risk.

   (a)   (*1 point*) Define each source.

ANSWER:

There are two sources of independent risk: parameter risk and process risk.

   (b)   (*1 point*) Define each source.

ANSWER:

You are provided the following information about the risks associated with these lines of business:

	Motor	Home
Percentage of Liabilities	60%	40%
Independent Risk CoV	8%	5%
Internal Systemic Risk CoV	4%	7%
External Systemic Risk CoV	4%	3%

The correlations between motor and home liabilities for each risk source are:

   - Independent Risk: 0%
   - Internal Systemic Risk: 50%
   - External Systemic Risk: 30%

   (c)   (*1.5 points*) Calculate the coefficient of variation for each risk source for both lines combined.

The response for part (c) is to be provided in the Excel spreadsheet.

   (d)   (*0.5 points*) Calculate the consolidated coefficient of variation from the three sources of uncertainty. Assume independence between each of the sources of uncertainty.

The response for part (d) is to be provided in the Excel spreadsheet.

7. (*4 points*) You are given the following quantities with respect to a retrospective rating plan:

- The expected loss, E, is 50.
- The total expenses, e, are 7.
- The loss conversion factor, C, is 1.125.
- The minimum premium, H, is 30.
- The maximum premium, G, is 75.

Table M		
r	$\phi(r)$	$\psi(r)$
0.00	1.00	0.00
0.20	0.81	0.01
0.40	0.64	0.04
0.60	0.49	0.09
0.80	0.36	0.16
1.00	0.25	0.25
1.20	0.16	0.36
1.40	0.09	0.49
1.60	0.04	0.64
1.80	0.01	0.81
2.00	0.00	1.00

- $G - H = CE(r_G - r_H)$
- $(e + E) - H = CE[\phi(r_H) - \phi(r_G)]$

(a) (*2 points*) Determine the values of r_H and r_G.

ANSWER:

(b) (*1 point*) Calculate the net insurance charge.

The response for part (b) is to be provided in the Excel spreadsheet.

(c) (*1 point*) Calculate the basic premium.

The response for part (c) is to be provided in the Excel spreadsheet.

8. (*5 points*) Specialist Reinsurance Company is offering finite reinsurance to Ceding Insurance Company to cover its aggregate annual losses.

The annual number of losses has a Poisson distribution with mean 1.5.

The loss size distribution is:

Loss Size (millions)	Probability
1	0.5
2	0.4
3	0.1

Loss sizes are independent of one another and independent of the number of losses.

(a) (*2 points*) Complete the following aggregate loss probability table:

Aggregate Losses (millions)	Probability
0	
1	0.1673
2	0.1966
3	0.1496
4	
5	
6	0.0411
7	
8	
9	
10	
11	
12	
13	
14	0.0001
15	0.0000

The response for part (a) is to be provided in the Excel spreadsheet.

The finite risk cover has the following terms:

- Annual Premium: 2.5 million
- Limit: 10 million
- Profit Commission: 80% after 10% margin on Annual Premium
- Additional Premium: 50% of (Loss + Margin − Annual Premium)

8. Continued

The underwriting results for different aggregate losses are:

Aggregate Losses (millions)	Underwriting Result (millions)
0	0.700
1	0.500
2	0.300
3	(0.125)
4	(0.625)
5	(1.125)
6	(1.625)
7	(2.125)
8	(2.625)
9	(3.125)
10 +	(3.625)

(b)   (*2 points*) Verify the following underwriting results for Specialist:

   (i)   A profit of 0.3 million if aggregate losses are 2 million.

   (ii)   A loss of 1.125 million if aggregate losses are 5 million.

   The response for part (b) is to be provided in the Excel spreadsheet.

(c)   (*0.5 points*) State the two conditions that a finite reinsurance arrangement must fulfill for a ceding company to consider it insurance.

   ANSWER:

(d)   (*0.5 points*) Explain whether the finite reinsurance can be considered insurance by Ceding Insurance Company.

   ANSWER

END OF EXAMINATION