GIRR Model Solutions
Spring 2022

Note: The numerical results shown in this document reflect the results of unrounded calculations. Candidates can reference the accompanying Excel file to see more decimal places.

1. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

Learning Outcomes:

(2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

(2d) Adjust historical earned premiums to current rate levels.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 11 and 12.

Commentary on Question:

This question tests the candidate's understanding of written premiums and adjusting premiums to current rate levels.

Solution:

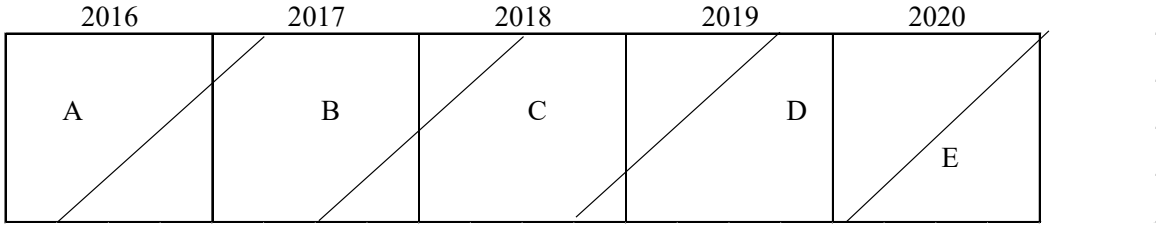
(a) Calculate the calendar year 2018 written premiums.

789,520 = 782,020 – 785,000 + 792,500

(b) Calculate the 2017, 2018, and 2019 on-level earned premiums, applicable for ratemaking, using the parallelogram method.

Commentary on Question:

The diagram is helpful to solve the question but not required for credit.



1. **Continued**

Level	Rate Level Index	Percent Premium Earned in Each Calendar Year at Rate Level 2017	2018	2019
A	1.00000	3.13%	-	-
B	1.02000	84.38%	12.50%	-
C	1.06080	12.50%	84.38%	28.13%
D	1.13506	-	3.13%	71.88%
E	1.16911	-	-	-
Total		100.0%	100.0%	100.0%

Average rate level in each CY:	1.02448	1.05802	1.11417
On-level factors for ratemaking:	1.1412	1.1050	1.0493

e.g., $1.02448 = 0.0313\times1.0 + 0.8438\times1.02 + 0.125\times1.608$
$1.1412 = 1.16911 / 1.02448$

Calendar Year	(1) On-Level Factor	(2) Earned Premiums	(3) = (1)(2) On-Level Earned Premiums
2017	1.1412	778,650	888,578
2018	1.1050	782,020	864,128
2019	1.0493	789,880	828,826

(c) Calculate the 2018 earned premium adjusted to current rate levels for ratemaking purposes for these two policies using the extension of exposures approach.

Policy	Months earned in 2018	% Earned in 2018	2018 Earned Premium	Future Rate Changes Oct. 1, 2018	Feb. 1, 2020	2018 On-Level Earned Premium
1	8	66.7%	3,333.33	7.0%	3.0%	3,673.67
2	2	16.7%	1,166.67	n/a	3.0%	1,201.67
			4,500.00			4,875.33

e.g., $3,333.33 = 5,000\times0.667$
$3,673.67 = 3,333.33\times1.07\times1.03$

1. Continued

(d) Explain why the answer in part (c) results in a different answer from multiplying the 2018 earned premiums for these two policies by the 2018 on-level factor calculated in part (b).

The parallelogram approach is an approximation method that assumes polices are written evenly throughout the year. These 2 polices do not represent policies that are written evenly (i.e., they are individual policies and not representative of the average). The extension of exposures approach is more accurate for individual policies.

(e) Critique this recommendation.

Recommend consistency, so adding the earned premiums from the 2 policies to the total earned premiums and then multiplying by the factor is recommended.

2. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(1d) Understand the components of ultimate values.
(2a) Create development triangles of claims and counts from detailed claim transaction data.
(3c) Identify the types of development triangles that can be used for investigative testing.
(3d) Analyze development triangles for investigative testing.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 3, 10 and 13.

Commentary on Question:

This question tests the candidate's understanding of creating a development triangle from detailed claims transaction data, and diagnostic tests that can be used on data triangles.

Solution:

(a) Update both development triangles shown above to include the claim transactions not captured due to the system error.

Accident	Incremental Reported Claims - Missing (000)			
Year	12	24	36	48
2018		15		20
2019	75		–10	
2020		65		
2021				

Accident	Cumulative Reported Claims - Missing (000)			
Year	12	24	36	48
2018	0	15	15	35
2019	75	75	65	
2020	0	65		
2021	0			

2. Continued

Accident	Reported Claims (000)			
Year	12	24	36	48
2018	1,196	1,540	1,653	1,758
2019	1,344	1,682	1,973	
2020	1,294	1,772		
2021	1,451			

e.g., 1,344 = 1,269 + 75

Accident	Incremental Reported Counts - Missing			
Year	12	24	36	48
2018		1		1
2019	1			
2020				
2021				

Accident	Cumulative Reported Counts - Missing			
Year	12	24	36	48
2018	0	1	1	2
2019	1	1	1	
2020	0	0		
2021	0			

Accident	Reported Counts			
Year	12	24	36	48
2018	230	251	261	267
2019	236	256	266	
2020	231	251		
2021	234			

(b) Determine calendar year 2021 reported claims.

Calendar year 2021 reported claims (000)
= (1,451+1,772+1,973+1,758) – (1,294+1,682+1,653) = 2,325

2. Continued

(c) Determine case reserves as of December 31, 2021, for accident year 2021 only.

Accident Year 2021 case reserves (000) = 1,451 – 800 = 651

(d) Describe the investigative tests you would recommend using for the following independent situations:

(i) The claim department implemented a new definition of claims to distinguish between reported incidents that are valid claims and incidents not covered under the insurance policy.

(ii) The claim department implemented a new initiative to increase their use of partial settlements.

(i) Ratios of closed no pay counts to closed counts

(ii) Any of the following is acceptable:
- Ratios of paid claims to reported claims
- Average paid claims (paid claims divided by closed counts)
- Average paid claims on closed with payment counts (paid claims divided by counts closed with payment)

(e) Provide two examples of company operational changes that could cause an increase in average reported claims without affecting reported counts.

Commentary on Question:
Other answers are possible.

Any two of the following are acceptable:
- Case reserve strengthening
- Increase in policy limits
- Expanded coverage
- Increase in defense costs, e.g., increased use of outside counsel

3. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6q) Distinguish occurrence-based and claims-made based coverage.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 34.

Commentary on Question:

This question tests the candidate's understanding of claims-made and occurrence policies.

Solution:

(a) Provide two reasons why AV might decide to purchase coverage.

- It offers protection against the possibility of a claim, especially considering the possibility of a claim from a past incident.
- Purchasing coverage also provides the opportunity to obtain coverage for the incident that could be a claim, provided the retroactive date of the policy is on or before the date of the incident.

(b) Recommend two exposure base options for XYZ to consider in providing insurance coverage. Justify your recommendations.

Commentary on Question:

Other answers are possible.

- Number of full-time equivalent professionals is a typical measure.
- Revenue could be used also because of increasing revenue.

(c) Provide one advantage and one disadvantage to AV in purchasing a *claims-made* policy.

- An advantage is that it is lower cost than an occurrence policy.
- A disadvantage is that nose or tail coverage may be required.

(d) Provide one advantage and one disadvantage to AV in purchasing an *occurrence* policy.

- An advantage is that it covers claims if occurrence coincides with policy period.
- A disadvantage is that it is more expensive than a claims-made policy, unless there is a charge for an old retroactive date.

4. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

Learning Outcomes:

(1q) Understand the types of reinsurance and key reinsurance terms.
(1s) Analyze and describe the types of reinsurance.
(1t) Understand important reinsurance contract provisions that potentially affect actuarial work.

Sources:

Fundamentals of General Insurance Actuarial Analysis 2019 Supplement, J. Friedland, Appendix H.

Commentary on Question:

This question tests the candidate's understanding of types of reinsurance contracts and determining the amounts paid by primary and reinsurance companies for various contract provisions.

Solution:

(a) Describe finite risk reinsurance.

The insurer pays an amount to the reinsurer to cover expected claims. If claims are less than expected, they receive a refund; if claims are more than expected the insurer pays an additional amount to the reinsurer.

(b) Explain why finite risk reinsurance has been controversial.

It can be seen as more of a loan than a transfer of risk. The Fitch Ratings Special Report said that companies are using it more to improve short term results, enhance capital or smooth earnings than to transfer risk.

4. Continued

(c) Determine the net amount paid by each company.

(i) Insurer A

(ii) Reinsurer B

Claims in Layer		Net Amount Paid by	
From	To	Insurer A	Reinsurer B
0	2,000,000	2,000,000	0
2,000,000	4,000,000	1,000,000	1,000,000
4,000,000	10,000,000	0	4,000,000
10,000,000		0	0
Total		3,000,000	5,000,000

Notes:
1. Insurer A covers the layer up to 2,000,000 attachment point
2. Claims in the layer 2,000,000 to 4,000,000 are shared 50%/50%
3. Reinsurer B covers the layer 4,000,000 to 10,000,000
4. Insurer A covers the layer excess of 10,000,000

(d) Calculate the total net amount paid by each company for this claim with pro-rata treatment of ALAE.

(i) Insurer A

(ii) Reinsurer B

ALAE % = 1,000,000 / 8,000,000 = 12.5%

	Net Amount Paid by	
	Insurer A	Reinsurer B
Claim	3,000,000	5,000,000
ALAE	375,000	625,000
Total	3,375,000	5,625,000

e.g., Insurer A pays 3,000,000×0.125 = 375,000 of the ALAE

4. Continued

(e) Calculate the total net amount paid by each company for this claim when ALAE is considered within the retention for this second claim.

(i) Insurer A

(ii) Reinsurer B

ALAE is within retention, so the total amount = claim & ALAE = 12,000,000 + 1,000,000 = 13,000,000.

Claims in Layer		Net Amount Paid by	
From	To	Insurer A	Reinsurer B
0	2,000,000	2,000,000	0
2,000,000	4,000,000	1,000,000	1,000,000
4,000,000	10,000,000	0	6,000,000
10,000,000		3,000,000	0
Total		6,000,000	7,000,000

Notes:
1. Insurer A covers the layer up to 2,000,000 attachment point
2. Claims in the layer 2,000,000 to 4,000,000 are shared 50%/50%
3. Reinsurer B covers the layer 4,000,000 to 10,000,000
4. Insurer A covers the layer excess of 10,000,000

5. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

7. The candidate will understand the need for monitoring results.

8. The candidate will be able to define an approach for actuarial analyses supporting financial reporting and ratemaking analyses under various real-life scenarios. The candidates will be able to define funding requirements for self-insurers.

Learning Outcomes:

(3f) Demonstrate knowledge of good practice related to projecting ultimate values.
(3g) Estimate ultimate values using the methods cited in (3e).
(4f) Calculate claim liabilities.
(7b) Analyze actual claims experience relative to expectations.
(8d) Estimate ultimate values.
(8e) Justify selections of ultimate values.
(8f) Develop reserves for financial reporting.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 14, 16, 17, 23, 36, and Appendices A-F.

Commentary on Question:

This question tests the candidate's understanding of estimating ultimate claims using the development method, expected method and the Bornhuetter Ferguson method. This question also tests the estimation of claim liabilities and the candidate's understanding of monitoring actual versus expected reported claims.

Solution:

5. Continued

(a) Calculate the ultimate claims for all accident years using the development method with paid claims. Justify any selections you make.

	Paid Claims Age-to-age factors						
AY	12-24	24-36	36-48	48-60	60-72	72-84	
2015	1.949	1.344	1.195	1.096	1.050	1.011	
2016	1.691	1.332	1.210	1.059	1.058		
2017	1.828	1.445	1.201	1.067			
2018	1.770	1.359	1.263				
2019	1.749	1.278					
2020	1.528						
Simple All	1.752	1.352	1.217	1.074	1.054	1.011	
Vol Wtd 5	1.704	1.349					
Vol Wtd All	1.737						
Medial All	1.810	1.361	1.198				Tail factor
Selected:	1.810	1.361	1.217	1.074	1.054	1.011	1.011

Rationale for selections:

- Medial all selected for 12-24 and 24-36 due to outliers
- Simple all years average selected thereafter
- Bondy method selected for tail factor as there was still development at 84 months

		Development Factors		Ultimate
AY	Paid Claims	Age-to-Age	Age-to-Ult.	Claims
2015	31,530	1.011	1.0106	31,866
2016	32,966	1.011	1.0214	33,671
2017	32,690	1.054	1.0765	35,189
2018	32,579	1.074	1.1561	37,665
2019	26,519	1.217	1.4071	37,315
2020	19,889	1.361	1.9148	38,083
2021	12,410	1.810	3.4648	42,999
Total	188,583			256,789

e.g., AY2017:

$1.0765 = 1.011 \times 1.011 \times 1.054$

$35{,}189 = 32{,}690 \times 1.0765$

5. Continued

(b) Calculate the trended on-level claim ratios for all accident years using the ultimate claims calculated in part (a).

Annual claim ratio trend = (1.05)(1 – 0.013) – 1 = 3.635%

AY	Earned Premiums	Premium On-Level Factors	Claim Trend @3.635%	Ultimate Paid Claims (000)	Trended On-Level Claim Ratio based on Paid Claims
2015	49,736,108	1.0722	1.2389	31,866	74.03%
2016	52,114,124	1.0681	1.1955	33,671	72.31%
2017	55,021,088	1.0420	1.1535	35,189	70.80%
2018	56,278,147	1.0265	1.1131	37,665	72.57%
2019	58,829,789	1.0182	1.0740	37,315	66.91%
2020	61,195,354	1.0092	1.0364	38,083	63.91%
2021	60,091,505	1.0000	1.0000	42,999	71.56%

e.g., 2017:

70.80% = (35,189×1.1535×1000)/(55,021,088×1.0420)

(c) Recommend a 2021 cost level expected claim ratio to use for estimating expected claims. Justify your recommendation.

	Trended On-Level Claim Ratio based on	
AY	Paid Claims	Reported Claims
2015	74.03%	76.80%
2016	72.31%	74.90%
2017	70.80%	73.80%
2018	72.57%	71.20%
2019	66.91%	77.70%
2020	63.91%	73.50%
2021	71.56%	79.40%
Average all years except 2021:	70.09%	74.65%
Average excluding high-low (except 2021):	70.65%	74.18%

Recommended claim ratio: 74.18%
Rationale: Recommend reported claim ratios as they seem more consistent. Exclude high and low to smooth fluctuations.

5. Continued

(d) Calculate expected claims for all accident years based on the recommendation in part (c).

AY	Earned Premiums	Premium On-Level Factors	Claim Trend @3.635%	Claim Ratio at Each AY Level	Expected Claims
2015	49,736,108	1.0722	1.2389	64.19%	31,927,613
2016	52,114,124	1.0681	1.1955	66.27%	34,537,640
2017	55,021,088	1.0420	1.1535	67.00%	36,866,223
2018	56,278,147	1.0265	1.1131	68.41%	38,497,891
2019	58,829,789	1.0182	1.0740	70.32%	41,369,001
2020	61,195,354	1.0092	1.0364	72.23%	44,202,496
2021	60,091,505	1.0000	1.0000	74.18%	44,572,874
Total					271,973,737

e.g., 2017:

67.00% = 74.18%×1.0420/1.1535

36,866,223 = 67.00%×55,021,088

(e) Calculate ultimate claims for all accident years using the Bornhuetter Ferguson method based on paid claims. Use the expected claims from part (d).

AY	Paid Claims (000)	Expected Claims	Age-to-Ultimate Development Factors	Ultimate Claims
2015	31,530	31,927,613	1.0106	31,866,187
2016	32,966	34,537,640	1.0214	33,689,509
2017	32,690	36,866,223	1.0765	35,308,413
2018	32,579	38,497,891	1.1561	37,777,546
2019	26,519	41,369,001	1.4071	38,488,222
2020	19,889	44,202,496	1.9148	41,006,787
2021	12,410	44,572,874	3.4648	44,118,543
Total	188,583			262,255,207

e.g., 2017: 35,308,413 = 32,690×1,000 + 36,866,223×(1 – 1/1.0765)

5. Continued

(f) Calculate the total unpaid claims for this line of business as of December 31, 3021, showing the case estimate and indicated IBNR separately.

Total reported claims: 238,061,000

Total unpaid claims = 271,794,051 – 188,583,000 = 83,211,051
Case estimate = 238,061,000 – 188,583,000 = 49,478,000
IBNR = 83,211,051 – 49,478,000 = 33,733,051

(g) Calculate the difference between the actual and expected reported claims for this line of business from December 31, 2021 through March 31, 2022 for all accident years, using linear interpolation.

	(1)	(2)	(3)	(4)	(5)
	As of Dec. 31, 2021				
AY	Selected Ultimate Claims	Reported Claims	Reported Claims at Mar. 31, 2022	Expected % Reported at Dec. 31, 2021	Expected % Reported at Mar. 31, 2022
2015	33,050,822	32,886,000	32,925,000	99.50%	99.63%
2016	34,902,242	34,555,000	34,599,600	99.01%	99.13%
2017	36,660,362	35,972,000	36,055,609	98.12%	98.34%
2018	37,986,078	35,453,000	36,105,780	93.33%	94.53%
2019	41,178,916	33,927,000	35,158,600	82.39%	85.12%
2020	42,698,643	31,041,000	32,342,000	72.70%	75.12%
2021	45,316,988	34,227,000	33,780,455	75.53%	74.82%
Total	271,794,051	238,061,000	240,967,044		

e.g., 2017:
(4): 98.12% = 36,660,362 / 35,972,000
(5): 98.34% = 98.12%×3/4 + 99.01%×1/4

5. Continued

	(6)	(7)	(8)
	Actual versus Expected Reported Claims from Dec. 31, 2021 through Mar. 31, 2022		
AY	Actual	Expected	Difference
2015	39,000	41,205	-2,205
2016	44,600	43,297	1,303
2017	83,609	80,907	2,702
2018	652,780	454,956	197,824
2019	1,231,600	1,126,481	105,119
2020	1,301,000	1,034,523	266,477
2021	-446,545	-320,630	-125,915
Total	2,906,044	2,460,739	445,305

e.g., 2017:

(6) = (3) – (2): 83,609 = 36,055,609 – 35,972,000

(7) = [(1) – (2)]×[(5) – (4)]/[1 – (4)]:

80,907 = (36,660,362 – 35,972,000)×(98.34% - 98.12%)/(1 – 98.12%)

(8) = (6) – (7): 2,702 = 83,609 – 80,907

(h) Provide an interpretation of the results for the actual versus expected analysis derived in part (g).

- Actual values are mostly significantly higher than expected, suggesting development factors are too low.
- 2021 actual value is much lower than expected, suggesting the development factor for 2021 is too high.

6. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6s) Explain the premise of experience rating.
(6t) Describe the types of experience rating used with general insurance.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 35.

Commentary on Question:

This question tests the candidate's understanding of individual risk rating.

Solution:

(a) Define the following terms in the context of individual risk rating:

(i) Prospective experience rating plan

(ii) Retrospective experience rating plan

(iii) Expense modification plan

(i) In a prospective experience rating program, the insurer adjusts the insured's future premiums, through discounts or surcharges, based on its claim experience in prior years.

(ii) In a retrospective experience rating program, the insured pays an initial deposit premium at the start of the policy term, and then, after the policy term is completed, retrospective refunds or surcharges are determined based on the actual claims during the policy term.

(iii) An expense modification plan is a form of rating plan (or rating procedure) where the variation of the premium for a particular insured is based on the variation in the expenses of the insurer with regard to this insured from those contemplated in the development of the manual rate.

(b) Provide one benefit of insurance company reliance on an insured's historical claims to project future claims for a prospective experience rating plan.

In relying on an insured's historical claims to project future claims, and in doing so to influence the determination of its premiums, the insurer provides incentives for the insured to manage its losses that result in claims to the insurer.

6. Continued

(c) Critique the use of a prospective experience rating plan for personal property coverage from an insurance company's perspective.

Commentary on Question:
Other answers are possible.

- It is difficult to hold insureds responsible as the cost would be significant from just one significant claim (volatility a concern).
- It would encourage risk control activities.
- With such low credibility, it is questionable that this would improve the predictive accuracy of premiums.

(d) Critique each characteristic in the new plan.

- Including only the most recent 3 years should improve responsiveness, but it might reduce credibility.
- Using a split rating formula will allow the plan to explicitly reflect the frequency and severity of an insured's experience.

(e) Explain why retrospective experience rating is typically not appropriate for each of the following:

(i) Insureds with low premium volume

(ii) Insureds with poor claims experience

(i) Insureds with small premium size are likely to have variable claims experience and one large claim may result in a maximum premium.

(ii) Insureds with poor claims experience will pay greater than the average premium and could have claims resulting in maximum premium.

7. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6l) Calculate risk classification changes.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 32.

Commentary on Question:

This question tests the candidate's understanding of classification ratemaking.

Solution:

(a) Determine if there is distributional bias in the exposure data. Support your conclusion.

Commentary on Question:

Only one inconsistency needs to be found to demonstrate distributional bias.

	Exposures			Ratios to territory 1 for each class:		
	Territory			Territory		
Class	1	2	3	1	2	3
A	2,700	2,700	2,025	1.00	1.00	0.75
B	1,350	2,025	2,700	1.00	1.50	2.00
C	1,350	675	4,050	1.00	0.50	3.00

e.g., Class A: 1.00 = 2,700/2,700; 1.00 = 2,700/2,700; 0.75 = 2,025/2,700

Since the ratios are not consistent for each class, there is distributional bias.

7. Continued

(b) Calculate the rebalanced pure premiums using the one-way analysis relativities for each rating variable combination.

			One-Way Relativities			
Class	Territory	Exposures	Class	Territory	Pure Premium	Rebalanced Pure Premium
A	1	2,700	0.8150	0.7567	213.94	209.93
A	2	2,700	0.8150	0.6522	184.40	180.94
A	3	2,025	0.8150	1.3637	385.57	378.34
B	1	1,350	0.9897	0.7567	259.80	254.92
B	2	2,025	0.9897	0.6522	223.92	219.72
B	3	2,700	0.9897	1.3637	468.21	459.43
C	1	1,350	1.2364	0.7567	324.54	318.45
C	2	675	1.2364	0.6522	279.72	274.48
C	3	4,050	1.2364	1.3637	584.89	573.92
Overall					353.53	346.90

e.g., Class A factor: 0.8150 = 282.73 / 346.90
Territory 3 factor: 1.3637 = 473.08 / 346.90
Class A, Territory 1 pure premium: 385.57 = 346.90×0.8150×1.3637
Overall pure premium: 353.53 = Sumproduct(exposures,pure premiums)
Class A, Territory 1 rebalanced pure premium:
378.34 = 385.57×353.53/346.90

(c) Calculate the revised relativities by class that result from a single iteration of the minimum bias method.

Commentary on Question:
Candidates need to start with one-way territory relativities to solve for class relativities.

One-way territory relativities :
- Territory 1: 262.50/346.90 = 0.7567
- Territory 2: 226.25/346.90 = 0.6522
- Territory 3: 473.08/346.90 = 1.3637

Total expected claims for each class:
- Class A: 240×2,700 + 200×2,700 + 450×2,025 = 2,099,250
- Class B: 270×1,350 + 250×2,025 + 450×2,700 = 2,085,750
- Class C: 300×1,350 + 260×675 + 500×4,050 = 2,605,500

7. Continued

First iteration for new class relativities using one-way territory relativities as starting point:

- Class A: 2,099,250/[(0.7567×2,700 + 0.6522×2,700 + 1.3637×2,025)×346.90] = 0.9217
- Class B: 2,085,750/[(0.7567×1,350 + 0.6522×2,025 + 1.3637×2,700)×346.90] = 0.9980
- Class C: 2,605,500/[(0.7567×1,350 + 0.6522×675 + 1.3637×4,050)×346.90] = 1.0753

(d) Describe the condition under which the converged results of the minimum bias method will be factors that reproduce all nine observed trended ultimate pure premiums.

The observed pure premiums must be independent for the minimum bias method to reproduce them.

8. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3c) Identify the types of development triangles that can be used for investigative testing.
(3d) Analyze development triangles for investigative testing.
(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.
(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 13 and 19.

Commentary on Question:

This question tests the candidate's understanding of Berquist-Sherman adjustments when there has been a change in case estimate adequacy and a change in claim settlement patterns.

Solution:

(a) Verify that the case estimates have increased for this line of business using one diagnostic test.

Change in average case is preferred as the ratios of paid to reported claims could be either due to a change in average case or a change in claim settlement patterns.

Accident	Average Case Estimates					
Year	12	24	36	48	60	72
2016	15,948	18,451	23,047	26,126	27,936	32,733
2017	16,881	19,537	24,087	27,664	32,429	
2018	17,816	20,541	25,486	32,125		
2019	18,881	21,761	29,339			
2020	19,690	25,185				
2021	22,360					

e.g., 2018 at 12 months: 17,816 = (38,734,090 – 10,407,100) / (3,391 – 1,801)

8. Continued

Accident	Change in Average Case Estimates				
Year	12	24	36	48	60
2016-2017	5.9%	5.9%	4.5%	5.9%	16.1%
2017-2018	5.5%	5.1%	5.8%	16.1%	
2018-2019	6.0%	5.9%	15.1%		
2019-2020	4.3%	15.7%			
2020-2021	13.6%				

e.g., 2018-2019 at 12 months: 6.0% = 18,881 / 17,816 – 1

There is a significant increase along the most recent diagonal which is evidence of an increase in case estimates.

(b) Describe a different diagnostic test from the test performed in part (a) that may indicate that case estimates have increased for this line of business.

The ratios of paid claims to reported claims could also indicate a possible change in case estimates.

- Calculate the triangle of paid claims to reported claims.
- In a stable environment, the values in each column should be consistent.
- A decrease in the ratios along the most recent diagonal could suggest a possible change in case estimates, however, a change in claim settlement pattern could also affect these ratios.

(c) Evaluate the disposal rates for this line of business to confirm that the rate of claims settlement has increased.

Disposal rates = ratios of closed counts to ultimate counts.

Accident	Disposal ratios					
Year	12	24	36	48	60	72
2016	0.534	0.669	0.771	0.782	0.809	0.950
2017	0.504	0.658	0.731	0.783	0.915	
2018	0.488	0.657	0.758	0.896		
2019	0.480	0.696	0.898			
2020	0.486	0.772				
2021	0.533					

The increase in the latest diagonal is evidence of the increase in claim settlement.

8. Continued

(d) Recommend disposal rates for each maturity age. Justify your recommendation.

Recommended disposal ratios: use the latest diagonal because that's where the rates increased.

12	24	36	48	60	72
0.533	0.772	0.898	0.896	0.915	0.950

(e) Calculate the adjusted case estimate triangle for this line of business, adjusting for changes in both case estimates and settlement rates. Justify any selections you make.

Adjusted Average Case = last diagonal from part (a), trended to each AY at 5%:

Accident	Adjusted Average Case Estimates					
Year	12	24	36	48	60	72
2016	17,520	20,719	25,344	29,138	30,884	*32,733*
2017	18,396	21,755	26,611	30,595	*32,429*	
2018	19,316	22,843	27,942	*32,125*		
2019	20,282	23,985	*29,339*			
2020	21,296	*25,185*				
2021	*22,360*					

e.g., 21,296 = 22,360 / 1.05

Adjusted Closed Counts:

- Latest diagonal from closed counts triangle
- Other values = selected disposal ratio × ultimate counts

Accident	Adjusted Closed Counts					
Year	12	24	36	48	60	72
2016	1,990	2,885	3,353	3,346	3,418	*3,548*
2017	1,988	2,882	3,350	3,343	*3,414*	
2018	1,967	2,851	3,314	*3,307*		
2019	1,975	2,863	*3,328*			
2020	1,975	*2,863*				
2021	*1,968*					

e.g., 2018 at 12 months: 1,967 = 0.533 × 3,691

8. **Continued**

Accident	Adjusted Open Counts = Reported Counts – Adjusted Closed Counts					
Year	12	24	36	48	60	72
2016	1,282	663	193	387	308	187
2017	1,287	631	258	350	308	
2018	1,424	619	296	364		
2019	1,296	554	248			
2020	1,369	614				
2021	1,322					

Adjusted Case Estimates = Adjusted Average Case Estimates × Adjusted Open Counts

Accident	Adjusted Case Estimates					
Year	12	24	36	48	60	72
2016	22,453,936	13,744,749	4,887,882	11,264,233	9,522,868	6,121,130
2017	23,671,034	13,737,756	6,877,722	10,713,283	9,987,990	
2018	27,506,953	14,148,080	8,280,886	11,693,510		
2019	26,275,582	13,287,871	7,275,990			
2020	29,143,944	15,463,340				
2021	29,560,500					

9. **Learning Objectives:**

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3k) Estimate ultimate claims by layer using common methods.

(3l) Understand the differences in development patterns and trends for various claim layers.

Sources:

Fundamentals of General Insurance Actuarial Analysis 2019 Supplement, J. Friedland, Appendix I.

Commentary on Question:

This question tests estimating ultimate claims and IBNR for various claim layers.

Solution:

(a) Explain why the theoretical approach might be the preferred approach.

There is more volatility in the excess layer. As a result, the excess layer has significantly more uncertainty.

(b) Calculate the IBNR for the layer 900,000 excess of 100,000 as of December 31, 2021 using theoretically-derived development factors at different limits.

	12	24	36	48	60
CDF 100,000	1.0534	1.0125	1.0078	1.0034	1.0010
CDF 1,000,000	1.0817	1.0127	1.0089	1.0040	1.0010

e.g., $1.0534 = 1.094 \times 0.622/0.646$

	(1)	(2)	(3)	(4)	(5) = (1)(3)	(6) = (2)(4)
	Reported Claims (000) as of December 31, 2021				Ultimate Claims (000) as of December 31, 2021	
Accident Year	Limit 100,000	Limit 1,000,000	CDF 100,000	CDF 1,000,000	Limit 100,000	Limit 1,000,000
2017	2,696	4,328	1.0010	1.0010	2,699	4,332
2018	2,816	4,112	1.0034	1.0040	2,826	4,128
2019	2,492	3,896	1.0078	1.0089	2,511	3,931
2020	3,185	4,784	1.0125	1.0127	3,225	4,845
2021	3,198	4,878	1.0534	1.0817	3,369	5,277
Total	14,387	21,998			14,629	22,513

9. Continued

	(7) = (2) – (1)	(8) = (6) – (5)	(9) = (8) – (7)
	Claims (000) in the Layer 900,000 Excess of 100,000		
Accident Year	Reported Claims	Ultimate Claims	IBNR (000)
2017	1,632	1,634	2
2018	1,296	1,303	7
2019	1,404	1,419	15
2020	1,599	1,620	21
2021	1,680	1,908	228
Total	7,611	7,884	273

10. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 29.

Commentary on Question:

This question tests the candidate's understanding of expenses used in ratemaking.

Solution:

(a) Describe the five major categories of expenses that are considered in a ratemaking analysis as defined by U.S. Standards.

Commentary on Question:

Candidates need to describe each expense in addition to simply listing the expenses.

- Loss adjustment expenses: expenses associated with investigating adjusting administering and settling claims
- Commission and brokerage fees: the compensation paid to agents and brokers for generating business
- Other acquisition expenses: all costs other than commissions and brokerage fees associated with the acquisition of business
- General administrative expenses: operational and administrative expenses (other than investment expenses)
- Taxes, licenses and fees: all taxes and miscellaneous fees except federal and foreign income taxes

(b) Describe two different ways for an insurer to incorporate non-proportional reinsurance in a ratemaking analysis.

1. Conduct the ratemaking analysis net of reinsurance excluding ceded premiums and ceded claims.
2. Conduct the ratemaking analysis on a gross of reinsurance basis and include the net cost of reinsurance as an expense.

10. Continued

(c) Describe the purpose of a residual market mechanism.

A residual market mechanism provides a means of obtaining coverage for individuals or organizations who are unable to secure insurance protection in the open market.

(d) Describe each of the following as used in U.S. workers compensation ratemaking:

(i) An expense constant

(ii) A premium discount plan

(i) A fixed/flat expense per policy for administrative costs that do not vary with premium.

(ii) A premium discount to recognize the administrative cost savings associated with larger insureds with higher premiums.

11. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 15.

Commentary on Question:

This question tests the candidate's understanding of the frequency-severity claim closure method for estimating unpaid claims.

Solution:

(a) Calculate the incremental closed counts for accident half-years 2021-1 and 2021-2 for all maturity ages.

Accident	Proportion of closed counts					
Half-Year	6	12	18	24	30	36
2019-1	0.5322	0.4602	0.3970	0.7007	0.9059	1.0000
2019-2	0.4841	0.4430	0.4101	0.7016	0.9005	
2020-1	0.5364	0.4675	0.3920	0.7016		
2020-2	0.4807	0.4358	0.3909			
2021-1	0.5357	0.4608				
2021-2	0.4849					
Averages:						
AHY-1	0.5348	0.4628	0.3945			
AHY-2	0.4832	0.4394	0.4005			
All	0.5090	0.4535	0.3975	0.7013	0.9032	1.0000
Selected:						
AHY-1	0.5348	0.4628	0.3945	0.7013	0.9032	1.0000
AHY-2	0.4832	0.4394	0.4005	0.7013	0.9032	1.0000

Incremental closed counts for 2021:							Ultimate Counts
2021-1	1,988	794	*367*	*394*	*152*	*16*	3,711
2021-2	1,848	*863*	*441*	*463*	*178*	*19*	3,811

11. Continued

Notes:

Proportion closed for 2020-2:

6 months: 0.4807 = 1,767 / 3,676

12 months: 0.4358 = 832 / (3,676 – 1,767)

18 months: 0.3909 = 421 / (3,676 – 1,767 – 832)

Incremental closed counts:

2021-1 at 18 months: 367 = 0.3945×(3,711 – 1,988 – 794)

2021-2 at 12 months: 863 = 0.4394×(3,811 – 1,848)

(b) Calculate the total unpaid claims for accident year 2021 as of December 31, 2021.

	Incremental Paid Severity					
AHY	6	12	18	24	30	36
2021-1	1,450.10	5,027.03	7,202.19	23,685.04	37,219.35	40,699.61
2021-2	1,500.00	5,200.00	7,450.00	24,500.00	38,500.00	42,100.00

	Projected Incremental Paid Claims					
AHY	6	12	18	24	30	36
2021-1			2,639,654	9,343,030	5,648,369	661,944
2021-2		4,485,468	3,283,480	11,334,239	6,852,164	803,019

Total unpaid claims for AY2021: 45,051,367

Notes:

Incremental paid severity for 2021-1 at 18 months:

$7{,}202.19 = 7{,}450 \times 1.07^{-0.5}$

Projected incremental paid claims for 2021-1 at 18 months:

2,639,654 = 7,202.19×367

12. Learning Objectives:

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

(4c) Evaluate and justify selections of unpaid unallocated loss adjustment expenses based on ratio and count-based methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 22.

Commentary on Question:

This question tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method, as well as the Kittel refinement with the Mango and Allen smoothing adjustment.

Solution:

(a) Estimate unpaid ULAE as of December 31, 2021, using the classical paid-to-paid method with a simple four-year average of historical experience, and a pure IBNR refinement.

Calendar Year	Paid ULAE	Actual Paid Claims	Ratio of Paid ULAE to Paid Claims
2018	16,172,450	176,261,530	9.18%
2019	16,807,540	184,338,130[(1)]	9.12%
2020	17,831,120	187,853,340	9.49%
2021	19,284,360	197,358,720	9.77%
Total	70,095,470	745,811,720	9.39%

Note: (1): 184,338,130 = 195,338,130 – 11,000,000 (adjustment for the large closed claim)

Unpaid ULAE = 9.39%×26,803,900 + 9.39%×(1 – 0.3)×(95,171,300 + 43,591,100) = 11,636,593

12. Continued

(b) Estimate unpaid ULAE as of December 31, 2021 using the Kittel refinement with the Mango and Allen smoothing adjustment, a simple four-year average of historical experience, and a pure IBNR refinement.

Calendar Year	(1) Paid ULAE	(2) Expected Paid Claims	(3) Expected Reported Claims	(4) = (1) / Avg[(2),(3)] ULAE Ratio
2018	16,172,450	181,712,920	179,693,890	8.95%
2019	16,807,540	188,100,130	190,637,250	8.88%
2020	17,831,120	195,680,570	206,174,180	8.87%
2021	19,284,360	205,582,000	222,977,380	9.00%
Total	70,095,470	771,075,620	799,482,700	8.92%

Unpaid ULAE = 8.92%×26,803,900 + 8.92%×(1 – 0.3)×(95,171,300 + 43,591,100) = 11,061,217

(c) Critique the appropriateness of each result from (a) and (b).

Since exposures are growing, the paid-to-paid ratio in part (b) will overstate ULAE, because the paid ULAE in ratio numerator will react to exposure growth faster than paid claims in ratio denominator.

The Kittel adjustment helps adjust for exposure growth and the Mango and Allen smoothing adjustment is useful for exposure growth.

The Mango and Allen smoothing adjustment is good for volatile lines (or lines with large claims, or lines with low-frequency, high-severity.

13. Learning Objectives:

9. The candidate will understand the nature and application of catastrophe models used to manage risks from natural disasters.

Learning Outcomes:

(9a) Describe the structure and modules of catastrophe models.
(9d) Understand and apply common risk metrics associated with catastrophe modeling results.
(9e) Understand the role of governance in catastrophe models.

Sources:

Uses of Catastrophe Model Output, American Academy of Actuaries, July 2018.

Commentary on Question:

This question tests the candidate's understanding of catastrophe modeling.

Solution:

(a) Explain why the 100-year PML for hurricane wind losses and the 100-year PML for tornado wind losses should not be added together to determine the 100-year PML for hurricane and tornado wind losses.

PMLs are not additive. This is because the probability that all causes have a one in 100-year event in the same year is much less than 1 percent. As such, the sum of the one in 100-year PMLs is associated with a much longer return period.

(b) Describe how an insurer could use each of the following loss metrics to understand the risk of an individual insured.

(i) AAL to TIV ratio

(ii) PML to TIV ratio

(i) AAL to TIV ratio: This ratio shows long-term risk at a location. It can be used to compare the long-term risk for properties that are close geographically.

(ii) PML to TIV ratio: This ratio gives an indication of possible loss severity at a location. It can be used to compare properties that have similar AALs to determine which one has a higher loss potential from extreme events.

13. Continued

(c) Explain how catastrophe models can be used by an insurer for portfolio optimization with respect to risk.

An insurer chooses a modeled metric that it considers important. It then builds a portfolio that optimizes that metric relative to a level of premium or exposure using outputs from catastrophe model runs.

(d) Provide two other examples of requirements that have been established to govern the use of catastrophe models.

Any two of the following are acceptable:

- The American Academy of Actuaries and insurance regulatory bodies have developed requirements and guidance for Actuaries in their development, use, and reliance on catastrophe models.
- Actuaries in the U.S. must follow ASOPs of which two are specifically focused on the use of catastrophe models.
- The State of Florida has a uses a legislated methodology for evaluating hurricane models that can be used.
- The NAIC in the U.S. requires model use for completion of RBC and ORSA.
- Enterprise Risk Management (ERM), rating agencies, and state insurance regulators mandate certain model output to be provided for use in evaluation of risk-bearing entities.

14. Learning Objectives:

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(4g) Describe the components of premium liabilities in the context of financial reporting.

(4h) Evaluate premium liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 24.

Commentary on Question:

This question tests the candidate's understanding of premium liabilities.

Solution:

(a) Verify that the following amounts are consistent with the written premiums provided:

(i) Calendar half-year 2021-1 gross earned premium of 510,927

(ii) Year-end 2021 gross unearned premiums of 515,716

(i) Calendar half-year 2021-1 gross earned premium

Calendar/ Accident Half Year	Written Premiums	% Earned in 2021-1	Earned Premiums 2021-1
2020-1	500,255	25.0%	125,064
2020-2	518,366	50.0%	259,183
2021-1	506,720	25.0%	126,680
Total			510,927

This value is consistent.

(ii) Year-end 2021 gross unearned premiums:

Written Premiums 2021-1	Written Premiums 2021-2	Total
126,680	389,036	515,716

This value is consistent.

14. Continued

(b) Recommend the expected claim ratio to be used in the determination of premium liabilities as of December 31, 2021. Justify your recommendation.

Calendar/ Accident Half Year	(1) Earned Premiums	(2) Ultimate Claims including ALAE	(3) = (2)/(1) Claim Ratio	(4) # of Years for Past Trend (years)	(5) = 1.01^(4) Claim Trend	(6) = (3)(5) Trended to 2021 Cost Level Claim Ratio Jan-Jun	(7) = (3)(5) Trended to 2021 Cost Level Claim Ratio July-Dec
2019-1	518,804	364,784	70.31%	2	1.0201	71.73%	
2019-2	520,827	232,393	44.62%	2	1.0201		45.52%
2020-1	514,671	365,518	71.02%	1	1.0100	71.73%	
2020-2	509,071	229,396	45.06%	1	1.0100		45.51%
2021-1	510,927	366,542	71.74%	0	1.0000	71.74%	
2021-2	512,630	233,315	45.51%	0	1.0000		45.51%
Total	3,086,930	1,791,948	58.05%			71.73%	45.51%

Unearned premiums at Dec. 31, 2021	Policies Written in 2021-1	Policies Written in 2021-2	Total
Earned in 2022-1	126,680	259,357	
Earned in 2022-2		129,679	
Total	126,680	389,036	515,716
Average accident dates in 2021:	2021-04-01	2021-10-01	
Average accident dates in 2022:			
Earned in 2022-1	2022-02-15	2022-04-01	
Earned in 2022-2		2022-08-15	
Claim trend factors:			
Earned in 2022-1	1.00878	1.00497	
Earned in 2022-2		1.00872	
Expected claim ratio			
Earned in 2022-1	72.36%	72.089%	
Earned in 2022-2		45.911%	
Weighted average expected claim ratio:		65.573%	

14. Continued

(c) Calculate the premium liabilities as of December 31, 2021, both gross and net of reinsurance.

		Gross	Net
(1)	Unearned premium reserve	515,716	386,787
(2)	Expected claim ratio	65.573%	65.573%
(3)	Expected claims = (1)(2)	338,173	253,629
(4)	Expected ULAE = $5.7\% \times (3)_{gross}$	19,276	19,276
(5)	Maintenance expenses = $30\% \times 18\% \times (1)_{gross}$	27,849	27,849
(6)	Total premium liabilities = (3) + (4) + (5)	385,297	300,754

15. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 12 and 18.

Commentary on Question:

This question tests the candidate's understanding of the Cape Cod method for estimating ultimate claims.

Solution:

(a) Describe one situation in which the Cape Cod method might be preferred over the Bornhuetter Ferguson method.

Any one of the following is acceptable:

- When the actuary wants to derive and expected value based on historical data (or an objective approach, or a specified formula) rather than an independent a priori estimate (or professional judgement).
- When the actuary wants to assume that the cost per exposure unit is constant for all years in the experience period.

(b) Describe one situation in which the Generalized Cape Cod method might be preferred over the Cape Cod method.

Any one of the following is acceptable:

- When the actuary wants to use a distinct expected claim ratio for each year in the experience period rather than a constant claim ratio for all years.
- When the actuary does not want to assume that the cost per exposure unit is constant for all years in the experience period.

15. Continued

(c) Calculate the adjusted expected claim ratio.

Accident Year (AY)	(1) Earned Premiums (000)	(2) Average Rate Level	(3) = $(2)_{2021}/(2)_{AY}$ Premium On-Level Factor	(4) = (1)(3) On-Level Earned Premium (000)	(5) Cumulative Development Factors
2013	29,614	1.0000	0.9849	29,167	1.011
2014	27,371	1.0000	0.9849	26,958	1.028
2015	27,077	0.9900	0.9948	26,938	1.049
2016	28,792	0.9800	1.0050	28,936	1.090
2017	30,307	0.9800	1.0050	30,459	1.159
2018	29,053	0.9800	1.0050	29,198	1.305
2019	26,785	0.9800	1.0050	26,919	1.709
2020	25,618	0.9800	1.0050	25,746	2.399
2021	27,616	0.9849	1.0000	27,616	3.999
Total	252,233			251,936	

Notes: Column (2) average rate levels:
AY2015: 0.99 = 0.5×1.0 + 0.5×0.98
AY2021: 00.9849 = 0.98×(7/8)+0.98×1.04×(1/8)

AY	(6) = 1 / (5) Expected % Developed	(7) = (4)(6) Used-Up On-Level Earned Premiums (000)	(8) Reported Claims as of Dec. 31, 2021 (000)	(9) = $1.02^{(2021-AY)}$ Claims Trend Factor	(10) = (8)(9) Adjusted Claims at Dec. 31, 2021 (000)
2013	98.9%	28,849	15,795	1.172	18,506
2014	97.3%	26,223	14,119	1.149	16,218
2015	95.3%	25,679	17,998	1.126	20,269
2016	91.7%	26,547	17,630	1.104	19,465
2017	86.3%	26,280	16,178	1.082	17,512
2018	76.6%	22,374	15,699	1.061	16,660
2019	58.5%	15,751	*11,231*	1.040	11,685
2020	41.7%	10,732	7,963	1.020	8,122
2021	25.0%	6,906	4,910	1.000	4,910
		189,342	121,523		133,347

Note: AY2019 Reported Claims (column (8)) excludes the 3,000,000 unusual claim that is not expected again (11,231 = 14,231 – 3,000).

15. Continued

Adjusted Expected Claim Ratio: 133,347 / 189,342 = 70.4%

(d) Calculate projected ultimate claims for all accident years.

AY	(11) = 70.4%×(4)/(9) Expected Claims (000)	(12) = 1 – (6) Expected % Unreported	(13) = (11)(12) Expected Unreported (000)	(14) = Reported Claims + (13) Projected Ultimate Claims (000)
2013	17,532	1.1%	191	15,986
2014	16,528	2.7%	450	14,569
2015	16,846	4.7%	787	18,785
2016	18,457	8.3%	1,524	19,154
2017	19,817	13.7%	2,719	18,897
2018	19,377	23.4%	4,529	20,228
2019	18,222	41.5%	7,560	21,791
2020	17,777	58.3%	10,367	18,330
2021	19,449	75.0%	14,585	19,495
	164,005		42,711	167,234

(e) Calculate expected claims for accident year 2021 using the Generalized Cape Cod approach and a decay factor of 80%.

AY	(7) Used-Up On-Level Earned Premiums (000)	(10) Adjusted Claims at Dec 31, 2021 (000)	(15) = (10) / (7) Claim Ratios	(16) = $0.8^{(2021-AY)}$ Decay Factors
2013	28,849	18,506	64.1%	16.8%
2014	26,223	16,218	61.8%	21.0%
2015	25,679	20,269	78.9%	26.2%
2016	26,547	19,465	73.3%	32.8%
2017	26,280	17,512	66.6%	41.0%
2018	22,374	16,660	74.5%	51.2%
2019	15,751	11,685	74.2%	64.0%
2020	10,732	8,122	75.7%	80.0%
2021	6,906	4,910	71.1%	100.0%
	189,342	133,347		

Expected claim ratio for AY2021:
sumproduct[(7),(15),(16)] / sumproduct[(7),(16)] = 71.8%
Expected claims for AY2021 = 71.8%×27,616×0.75 + 4,910 = 19,771

16. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(5b) Identify the time periods associated with trending procedures.
(5c) Analyze and evaluate trend for claims (including frequency, severity, and pure premium) and exposures (including inflation-sensitive exposures and premiums).
(5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
(5e) Calculate trend factors for claims and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 26.

Commentary on Question:

This question tests the candidate's understanding of premium trend and adjusting premiums for trend for ratemaking purposes.

Solution:

(a) Calculate the percentage increase in premiums that occurred from the rating differentials change on July 1, 2021.

Weighted average differentials using rates prior to July 1, 2021 = 1.0000125
Weighted average differentials using rates effective July 1, 2021 = 1.0025387
(i.e., weighted averages use 2021 earned exposures)

Estimated percent premium change from differential change:
1.0025387 / 1.0000125 – 1 = 0.25%

(b) Recommend the annual premium trend rate to use for ratemaking for this line of business. Justify your recommendation.

	2017	2018	2019	2020	2021
Weighted average differential (using July 1, 2021 differentials)	0.9881932	0.9935602	0.9965078	0.9994956	1.0025387
Year-to-year change e.g., 0.9935602 / 0.9881932 – 1 = 0.54%		0.54%	0.30%	0.30%	0.30%

Recommended annual trend: 0.30%
Justification: Annual change has stabilized at 0.3% over the last 3 years, so it is reasonable to assume that trend will continue into the future rating period.

16. Continued

(c) Calculate the calendar year 2021 earned premiums to use for ratemaking.

Average earned date in future rating period for 12-month policies:	Oct. 1, 2023
Trending period (months) for 12-month policies: July 1, 2021 to Oct. 1, 2023:	27
Average earned date in future rating period for 6-month policies:	Jul. 1, 2023
Trending period (months) for 12-month policies: July 1, 2021 to Jul. 1, 2023:	24
Trending period (months) weighted by policy term (27×2/3 + 24×1/3)	26
Trend factor = $(1 + 0.003)^{(26/12)}$ =	1.00651834
Trended premium for ratemaking = 25,256,000 × 1.00651834 =	25,420,627

17. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(5b) Identify the time periods associated with trending procedures.
(5e) Calculate trend factors for claims and exposures.
(6g) Calculate loadings for catastrophes and large claims.
(6h) Apply loadings for catastrophes and large claims in ratemaking.
(6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.
(6k) Demonstrate the use of credibility in ratemaking.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 26, 30, and 31.

Commentary on Question:

This question tests the candidate's understanding of basic ratemaking, including the application of a loading for wildfire claims in ratemaking.

Solution:

(a) Calculate the ultimate pure premium for wildfire claims to be used as a loading in the homeowners premiums.

Average accident date in future rating period: Sep. 1, 2023

	(1)	(2)	(3)	(4)	(5)
		Wildfire - Ultimate			
Accident Year	Earned Exposures	Counts	Claims	Trending Period (years)	Severity Trend @3%
2015	11,200	0	0	8.167	1.2730
2016	11,850	0	0	7.167	1.2359
2017	12,500	1	1,500,000	6.167	1.1999
2018	13,750	0	0	5.167	1.1650
2019	15,000	1	1,120,000	4.167	1.1311
2020	16,250	0	0	3.167	1.0981
2021	17,500	1	500,000	2.167	1.0661
Total	98,050	3	3,120,000		

17. Continued

Accident Year	(6) = (3)(5) Trended Ultimate Claims	(7) = (2)/(1) Trended Ultimate Wildfire Frequency	(8) = (6)/(2) Trended Ultimate Wildfire Severity	(9) = (6)/(1) Trended Ultimate Wildfire Pure Premium
2015	0	0.000000	0	0.00
2016	0	0.000000	0	0.00
2017	1,799,924	0.000080	1,799,924	143.99
2018	0	0.000000	0	0.00
2019	1,266,795	0.000067	1,266,795	84.45
2020	0	0.000000	0	0.00
2021	533,070	0.000057	533,070	30.46
Total	3,599,789	0.000031	1,199,930	36.71

(b) Calculate the indicated total premium for the homeowners coverage, including a loading for wildfire claims.

	Credibility	Trended Ultimate Pure Premium
Insurer internal experience from part (a)	20%	36.71
Industry experience	80%	50.00
Credibility weighted wildfire claims experience (at Sept. 1, 2023 cost level): 0.2×36.71 + 0.8×50.00		47.34
Non-wildfire claims per policy (PP) as of July 1, 2021: 21,507,500×0.67/17,500 =		823.43
Trended non-wildfire PP to future rating period = 823.43×[(1 + 0.04)/(1 + 0.025)]$^{2.167}$ =		849.76
Indicated premium = (849.76 + 47.34 + 70) / (1 – 0.2 – 0.05) =		1,289.47

18. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3h) Explain the effect of changing conditions on the projection methods cited in (3e).
(3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.
(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapters 20 and 21.

Commentary on Question:

This question tests the candidate's understanding of effect that changing conditions have on the estimates of ultimate claims.

Solution:

(a) Describe a data substitution that you would make in your analysis to mitigate the problem for each of the following independent scenarios.

(i) There is a change in policy limits between successive policy years.

(ii) Exposure growth during the past two years has caused a distortion in recent development factors due to significant shifts in the average accident date within each accident year.

(iii) A tort reform change two years ago reduced the expected severity of many newly reported claims.

(iv) There has been a change in the definition of claim count you typically use for diagnostics.

(i) Substitute policy year data for accident year data.

(ii) Substitute accident quarter data for accident year data.

(iii) Substitute report year data for accident year data.

(iv) Substitute earned exposures in place of claim counts.

18. Continued

(b) Describe the effect you expect this shift to have on an accident year claim triangle using reported claims.

Since liability claims have a longer reporting tail than property claims, I expect to see an increase in development at later evaluations in the triangle.

(c) Describe an approach to estimating ultimate claims for this business.

Use a frequency-severity method and explicitly address the changing liability severity.

19. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6m) Describe key considerations in the analysis of deductible factors and increased limits factors.

(6n) Calculate deductible factors and increased limits factors.

Sources:

Fundamentals of General Insurance Actuarial Analysis, J. Friedland, Chapter 33.

Commentary on Question:

This question tests the candidate's understanding of deductible factors.

Solution:

(a) Calculate the indicated deductible factor for a deductible of 1,000.

Indemnity eliminated at 500 deductible:	
886,650 + 7,070 × 500	4,421,650
Total Indemnity at 500 deductible:	
12,605,205 – 4,421,650	8,183,555
Indemnity eliminated at 1,000 deductible:	
886,650 + 1,976,260 + 4,210 × 1,000	7,072,910
Total Indemnity at 1,000 deductible:	
12,605,205 – 7,072,910	5,532,295
Deductible relativity: 5,532,295 / 8,183,555	0.676

(b) Recommend a factor for a deductible of 1,500. Justify your recommendation.

First need to know the 2,000 deductible factor:
Indemnity eliminated at 2,000 deductible:

886,650 + 1,976,260 + 3,256,395 + 1,975 × 2,000 = 10,069,305

Total Indemnity at 2,000 deductible: 12,605,205 – 10,069,305 = 2,535,900

Deductible relativity = 2,535,900 / 8,183,555 = 0.310

Therefore, relativity needs to be between 0.676 and 0.310 → can use consistency test to find the appropriate range for a factor.

19. Continued

Relativity for 1,500 deductible = x
Based on consistency test,
Difference between 500 & 1000, and 1000 & 1500: $1 - 0.676 > 0.676 - x$
(note: can ignore denominators since all are 500)
solves for $x > 0.352$

Difference between 1000 & 1500, and 1500 & 2000: $0.676 - x > x - 0.310$
solves for $x < 0.493$

Therefore, recommend any factor higher than 0.352 and lower than 0.493.

(c) Describe why you would not be able to use data from policies with a 2,000 deductible to determine the deductible factor for a 1,000 deductible if the data was censored.

There may have been claims for amounts between 1000 and 2000 that we don't know about, and we would need to include those claims in the calculation.

(d) Provide a reason why you would choose to determine deductible factors using a classification ratemaking approach instead of using the elimination ratio approach.

Claimants' behavior and claim experience may differ between different deductibles.